

TRANSPORT OF INTERACTING ELECTRONS THROUGH A PAIR OF COUPLED METALLIC QUANTUM DOTS

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We derive a complete expression for the interaction correction to the $I - V$ curve of two connected in series metallic quantum dots. For strongly asymmetric dots in a wide range of parameters this interaction correction depends logarithmically on voltage and temperature.

1 Introduction

Recently it was demonstrated¹ that important information about the effect of electron-electron interactions on quantum transport in disordered conductors can be obtained within a transparent theoretical framework of quasiclassical Langevin equations generalized to situations in which relaxation of the electron distribution function occurs at much longer time scales as compared to the electron dwell time between two neighboring scatterers. The main goal of this paper is to employ the approach¹ in order to study interaction effects in low temperature electron transport through the system of three quantum scatterers or, equivalently, two coupled quantum dots. This structure can serve as a model for a number of transport experiments. In addition to that, the system of three scatterers appears to be the simplest one in which the interaction correction to the $I - V$ curve is described by two different contributions, one of them being absent in a yet simpler system of two quantum scatterers².

2 General Results

Let us briefly recollect our results¹ for the electron-electron interaction correction to the current in a chain of N arbitrary coherent scatterers (with resistances R_n) or $N - 1$ quantum dots. Assuming that the dimensionless conductances of the scatterers $g_n = 2\pi/e^2 R_n = 2 \sum_k T_k^{(n)}$ are large (here and below $T_k^{(n)}$ denotes the transmission of the k -th conducting mode in the n -th scatterer), $g_n \gg 1$, one can effectively expand the general expression for the current in the inverse conductance and find

$$I = \frac{V}{R_\Sigma} + \delta I_1 + \delta I_2, \quad (1)$$

where¹

$$\begin{aligned} \delta I_1 = & \frac{1}{4eR_\Sigma} \sum_{n,m=1}^N g_m \int dt dt' \frac{T^2 \sin eVt}{\sinh^2 \pi Tt} K_{mn}(t-t') [\delta_m(a_{m-1} - a_{n-1}) D_{n-1,m}(t') - \delta_{m-1} \\ & \times (a_m - a_{n-1}) D_{n-1,m-1}(t') + \delta_m(a_{m-1} - a_n) D_{n,m}(t') - \delta_{m-1}(a_m - a_n) D_{n,m-1}(t')] \end{aligned} \quad (2)$$

$$\begin{aligned} \delta I_2 = & -\frac{\pi}{eR_\Sigma^2} \sum_{n,m=1}^N \beta_n R_n \int dt dt' \frac{T^2 \sin eVt}{\sinh^2 \pi Tt} K_{mn}(t-t') \left[\delta_{nm} \delta(t') - \frac{g_m}{4\pi} (\delta_{m-1} D_{n-1,m-1}(t') \right. \\ & \left. - \delta_m D_{n-1,m}(t') - \frac{g_m}{4\pi} (\delta_m D_{nm}(t') - \delta_{m-1} D_{n,m-1}(t')) \right]. \end{aligned} \quad (3)$$

Here V is the voltage bias applied to the chain of quantum dots, $R_\Sigma = \sum_{n=1}^N R_n$ is the total chain resistance, $\beta_n = \sum_k T_k^{(n)}(1 - T_k^{(n)}) / \sum_k T_k^{(n)}$ is the Fano factor of the n -th scatterer, δ_n stands for the mean level spacing in the n -th dot δ_n and $a_n = \sum_{j=1}^n R_j / R_\Sigma$.

The diffuson $D_{n,m}(t)$ satisfies the equation

$$\frac{\partial D_{n,m}}{\partial t} = \frac{\delta_n}{4\pi} (g_n D_{n-1,m} + g_{n+1} D_{n+1,m} - (g_n + g_{n+1}) D_{n,m}) + \delta_{nm} \delta(t) \quad (4)$$

with the boundary conditions

$$D_{0,m} = D_{N,m} = D_{n,0} = D_{n,N} = 0. \quad (5)$$

The function $K_{nm}(t)$ is defined by the formula

$$K_{nm}(t) = e^2 \int \frac{d\omega}{2\pi} e^{-i\omega t} \frac{Z_{nm}(\omega)}{-i\omega + 0}, \quad (6)$$

where the effective impedance $Z_{nm}(\omega)$ is determined by the following set of equations

$$\begin{aligned} \delta q_n &= C_{n+1} Z_{n+1,m} - C_n Z_{nm} - C_{gn} \sum_{j=1}^n Z_{jm} \\ -i\omega \delta q_n &= \frac{Z_{nm}}{R_n} - \frac{Z_{n+1,m}}{R_{n+1}} + \frac{g_n \delta_{n-1}}{4\pi} \delta q_{n-1} + \frac{g_{n+1} \delta_{n+1}}{4\pi} \delta q_{n+1} \\ &\quad - \frac{(g_n + g_{n+1}) \delta_n}{4\pi} \delta q_n - \delta_{nm} + \delta_{n+1,m}, \\ \sum_{n=1}^N Z_{nm} &= 0. \end{aligned} \quad (7)$$

Here we introduced the capacitances of the n -th scatterer and the n -th dot, respectively C_n and C_{gn} , as well as the Kronecker symbols δ_{nm} .

3 Two quantum dots

In Ref. 1 we have explored the above general results for a chain of $N - 1$ identical quantum dots connected by N identical scatterers. In that case Eqs. (4), (7) can be resolved for arbitrary number of scatterers N . Here we restrict the number of scatterers to $N = 3$, i.e. consider transport of interacting electrons through a pair of coupled quantum dots. As we have already pointed out, this system appears to be the simplest one in which both interaction corrections δI_1 and δI_2 differ from zero. Below we will establish explicit analytical expressions for both these corrections and analyze them in several important limiting cases.

In the case of a chain with only three scatterers $D_{nm}(t)$ is described by a 2×2 matrix. Defining the inverse electron dwell times for both quantum dots, $1/\tau_1 = (g_1 + g_2)\delta_1/4\pi$ and $1/\tau_2 = (g_2 + g_3)\delta_2/4\pi$, and performing the Fourier transformation one arrives at the equation

$$\begin{pmatrix} -i\omega + 1/\tau_1 & -\delta_1 g_2/4\pi \\ -\delta_2 g_2/4\pi & -i\omega + 1/\tau_2 \end{pmatrix} \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (8)$$

which can be easily resolved. Analogously, the impedance matrix Z_{nm} has a 3×3 structure and it can be directly evaluated as well. Substituting the corresponding results into Eqs. (3), (2) we obtain

$$\delta I_1 = -2eT \frac{R_1 R_2 R_3}{R_\Sigma^3} \frac{W\left(\frac{\gamma_2 + ieV}{2\pi T}\right) - W\left(\frac{\gamma_1 + ieV}{2\pi T}\right)}{\sqrt{1 - \frac{4\delta_1 \delta_2 R_1 R_2 R_3 R_\Sigma}{[\delta_1 R_2 R_3 + \delta_2 R_1 R_2 + (\delta_1 + \delta_2) R_1 R_3]^2}}}, \quad (9)$$

$$\begin{aligned}
\delta I_2 = & -\frac{2eT}{R_\Sigma^2(C_{\Sigma 1}C_{\Sigma 2} - C_2^2)} \left\{ \mathcal{B}_1 \frac{W\left(\frac{\kappa_2 + ieV}{2\pi T}\right) - W\left(\frac{\kappa_1 + ieV}{2\pi T}\right)}{\kappa_2 - \kappa_1} \right. \\
& + \mathcal{B}_2 \left[\frac{\kappa_2 W\left(\frac{\kappa_1 + ieV}{2\pi T}\right) - \kappa_1 W\left(\frac{\kappa_2 + ieV}{2\pi T}\right)}{\kappa_1 \kappa_2 (\kappa_2 - \kappa_1)} - \frac{\gamma_2 W\left(\frac{\gamma_2 + ieV}{2\pi T}\right) - \gamma_1 W\left(\frac{\gamma_1 + ieV}{2\pi T}\right)}{\kappa_1 \kappa_2 (\gamma_2 - \gamma_1)} \right] \\
& \left. + \mathcal{B}_3 \frac{W\left(\frac{\gamma_2 + ieV}{2\pi T}\right) - W\left(\frac{\gamma_1 + ieV}{2\pi T}\right)}{\kappa_1 \kappa_2 (\gamma_2 - \gamma_1)} \right\}, \tag{10}
\end{aligned}$$

Here we defined $W(x) = \text{Im}[x\Psi(1+x)]$, where $\Psi(x)$ is the digamma function. We also defined $C_{\Sigma 1} = C_1 + C_2 + C_{g1}$, $C_{\Sigma 2} = C_2 + C_3 + C_{g2}$ as well as

$$\begin{aligned}
\mathcal{B}_1 &= C_{\Sigma 1}\beta_1 R_1 + C_{\Sigma 2}\beta_3 R_3 + (C_{\Sigma 1} + C_{\Sigma 2} - 2C_2)\beta_2 R_2, \\
\mathcal{B}_2 &= \beta_1 \frac{R_1(R_2 + R_3)}{R_2 R_3} + \beta_2 \frac{R_2(R_1 + R_3)}{R_1 R_3} + \beta_3 \frac{R_3(R_1 + R_2)}{R_1 R_2}, \\
\mathcal{B}_3 &= \left(\left(\frac{1}{R_2} + \frac{1}{R_3} \right) \frac{1}{\tau_2} + \frac{\delta_1 g_2}{4\pi} \frac{1}{R_2} \right) \beta_1 R_1 \\
&+ \left(\left(\frac{1}{R_1} + \frac{1}{R_2} \right) \frac{1}{\tau_1} + \frac{\delta_2 g_2}{4\pi} \frac{1}{R_2} \right) \beta_3 R_3 + \left(\frac{1}{R_1} \frac{\delta_1 g_1}{4\pi} + \frac{1}{R_3} \frac{\delta_2 g_3}{4\pi} \right) \beta_2 R_2
\end{aligned}$$

and

$$\begin{aligned}
\kappa_{1,2} &= \frac{\nu \mp \sqrt{\nu^2 - 4(C_{\Sigma 1}C_{\Sigma 2} - C_2^2) \left(\frac{1}{R_1 R_2} + \frac{1}{R_1 R_3} + \frac{1}{R_2 R_3} \right)}}{2(C_{\Sigma 1}C_{\Sigma 2} - C_2^2)}, \\
\nu &= \frac{C_{\Sigma 1}}{R_3} + \frac{C_{\Sigma 2}}{R_1} + \frac{C_{\Sigma 1} + C_{\Sigma 2} - 2C_2}{R_2}, \\
\gamma_{1,2} &= \frac{1}{2} \left(\frac{1}{\tau_1} + \frac{1}{\tau_2} \right) \mp \frac{1}{2} \sqrt{\left(\frac{1}{\tau_1} - \frac{1}{\tau_2} \right)^2 + \frac{\delta_1 \delta_2 g_2^2}{4\pi^2}}.
\end{aligned}$$

The above equations fully determine the leading interaction correction to the current in a system of two coupled quantum dots. These equations can now be analyzed in various physical limits. For the case of identical scatterers and dots our results match with those derived in Ref. 1, while in the limit $R_2 = 0$ the problem is reduced to that of a single quantum dot².

Let us first restrict our attention to the case of fully open quantum dots $\beta_{1,2,3} = 0$. In this case the term (10) vanishes identically, $\delta I_2 \equiv 0$, and the effect of electron-electron interactions on the $I - V$ curve is described only by the β -independent term δI_1 (9). At $T \rightarrow 0$ for the differential conductance we obtain

$$\frac{dI}{dV} = \frac{1}{R_\Sigma} - \frac{e^2}{2\pi} \frac{R_1 R_2 R_3}{R_\Sigma^3 \sqrt{1 - \frac{4\delta_1 \delta_2 R_1 R_2 R_3 R_\Sigma}{[\delta_1 R_2 R_3 + \delta_2 R_1 R_2 + (\delta_1 + \delta_2) R_1 R_3]^2}}} \ln \frac{\gamma_2^2 + e^2 V^2}{\gamma_1^2 + e^2 V^2}. \tag{11}$$

This equation demonstrates that at large voltages $eV \gg \gamma_2$ the interaction correction remains small while for $eV < \gamma_1$ it saturates to a finite voltage-independent value, i.e. at such voltages the $I - V$ curve of our system is Ohmic. In the limit $\gamma_1 \ll \gamma_2$ the interaction correction depends logarithmically on voltage provided the condition $\gamma_1 \ll eV \ll \gamma_2$ is fulfilled.

Assume now that one of the two dots, e.g. the second one, is very large. In this case both δ_2 and $1/\tau_2$ remain small. Setting $\delta_2 \rightarrow 0$, for $eV\tau_1 \ll 1$ from (11) one finds

$$\frac{dI}{dV} = \frac{1}{R_\Sigma} \left(1 + \frac{2}{g_\Sigma} \frac{R_1 R_2 R_3}{R_\Sigma^3} \ln(eV\tau_1) \right), \tag{12}$$

where $g_\Sigma = 2\pi/e^2 R_\Sigma$. For $\delta_2 = 0$ this result remains valid down to exponentially low voltages.

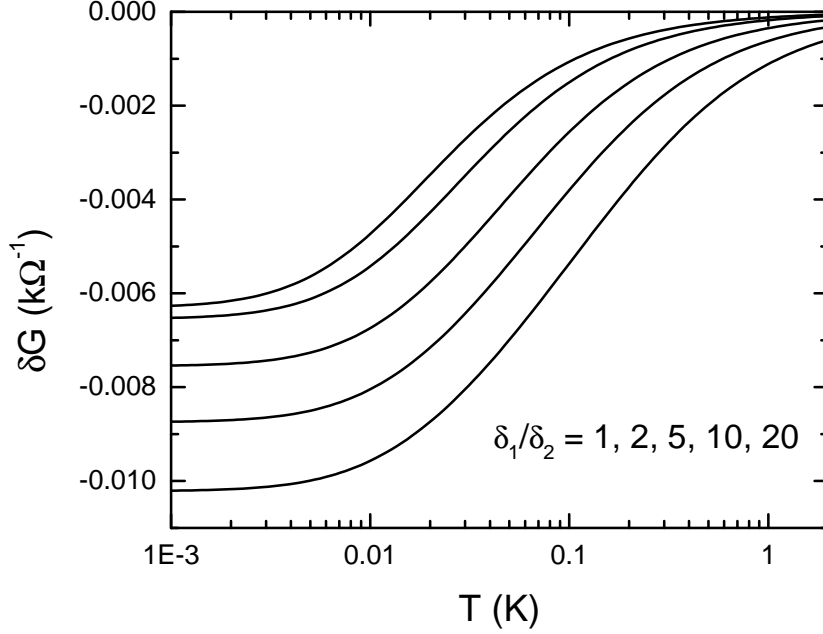


Figure 1: The temperature dependent interaction correction $\delta G(T)$ to the linear conductance for a pair of open quantum dots ($\beta_{1,2,3} = 0$) with $R_{1,2,3} = 1 \, \Omega$ and $\delta_2 = 1.5 \, \text{mK}$. The ratio δ_1/δ_2 increases from top to bottom.

Now let us briefly consider a general case of partially transparent quantum dots $\beta_{1,2,3} \neq 0$. In this case the term δI_2 (10) differs from zero and should be added to the correction δI_1 studied above. Provided the asymmetry between two dots is not large, at low enough voltages $eV < \gamma_1$ the term δ_1 saturates to a finite value similarly to δI_1 . For $\delta_2 \rightarrow 0$ and $eV\tau_1 \ll 1$ the correction δ_1 depends logarithmically on voltage (cf. Eq. (12)). Adding both δI_1 and δI_2 together and assuming $R_3 \ll R_1, R_2$, for $eV\tau_1 \ll 1$ and $T \rightarrow 0$ we obtain

$$\frac{dI}{dV} = \frac{1}{R_1 + R_2} \left(1 + \frac{2\tilde{\beta}_{12}}{g_3} \ln(eV\tau_1) \right) + \delta G, \quad (13)$$

where δG is the negative voltage-independent contribution (not presented here) originating from energies above $1/\tau_1$ and

$$\tilde{\beta}_{12} = \frac{R_1^2(R_2 + R_1\beta_1) + R_2^2(R_1 + R_2\beta_2)}{(R_1 + R_2)^3}$$

is the total Fano factor^{1,2} for the first quantum dot. The result (13) matches with one obtained in Ref. 2 for a single quantum dot shunted by an Ohmic resistor. In our case the role of such a shunt is played by a large quantum dot with $\delta_2 \rightarrow 0$.

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